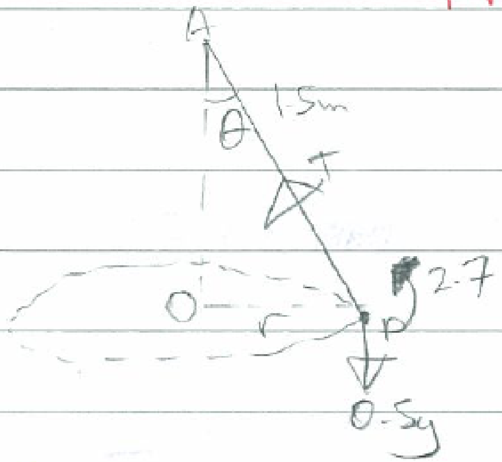
M3 - January 2005

1.



a) $\cancel{0.5g = T\cos\theta}$

$\leftarrow$ $\boxed{F = ma}$

$T\sin\theta = 0.5 \cdot 2.7^2 \cdot 1.5\sin\theta$

$T = 5.47N$ (3sf)

b) $\uparrow$ $0.5g = T\cos\theta$

$$\cos\theta = \frac{0.5 \times 9.8}{5.47}$$

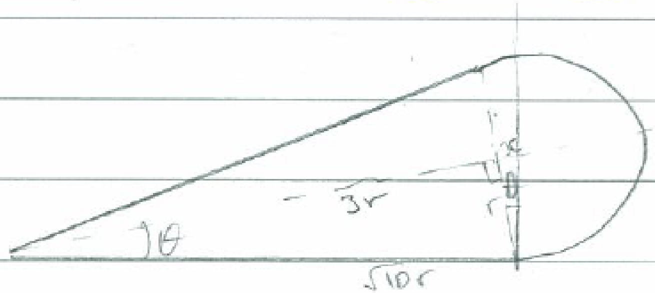
$\theta = 26°$ (2sf)

2. a) $M \times \frac{3}{8}r - m \times \frac{3}{4}r = (M+m)\bar{x}$

$$\frac{3Mr - 6mr}{8(M+m)} = \bar{x}$$

$$d = \frac{3(m-2m)}{8(m+m)}r$$

b)



$$\tan\theta = \frac{r}{3r} = \frac{1}{3}$$

$$\frac{1}{3} = \frac{x}{r} \quad x = \frac{r}{3}$$

$$d > x$$

$$\frac{3(m-2m)}{8(m+m)}r > \frac{r}{3}$$

$$\frac{3m-6m}{8m+8m} > \frac{1}{3}$$

$$9m - 18m > 8m + 8m$$

$$m > 26m$$

3. a) $\int y \, dx \, \bar{y} = \int_0^{\pi} y \, dx \times \bar{y}$ $\quad \frac{dy}{dx} = \cos x$

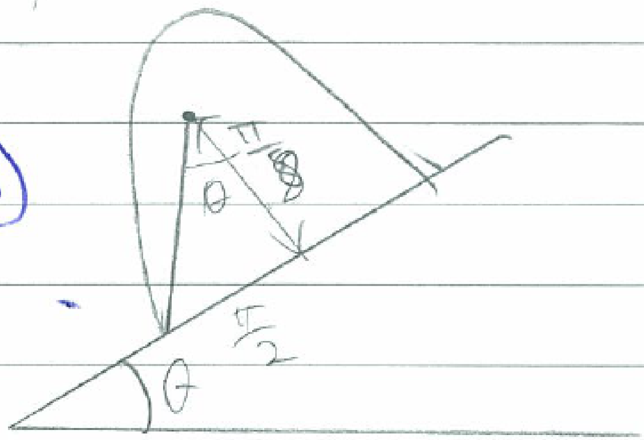
$\frac{1}{2}\int_0^{\pi} \cancel{\ldots} \sin^2 x \, dx = \bar{y}\int_0^{\pi} \sin x \, dx$

$\frac{1}{4}\int_0^{\pi} \frac{1-\cos 2x}{\cancel{\ldots}} \, dx = \bar{y}\left[-\cos x\right]_0^{\pi} = 2\bar{y}$

~~let $u = x$ $\quad \frac{dv}{dx} = \sin 2x$ $\quad \frac{du}{dx} = 1$ $\quad v = -\frac{1}{2}\cos 2x$~~

$\bar{y} = \frac{1}{8}\int_0^{\pi} 1-\cos 2x \, dx = \frac{1}{8}\left[x - \frac{1}{2}\sin 2x\right]_0^{\pi}$

$= \frac{1}{8} \times \pi = \frac{\pi}{8}$

b) 

$\tan\theta = \frac{\pi}{2} \div \frac{\pi}{8} = \frac{\pi}{2} \times \frac{8}{\pi} = 4$

$\theta_{max} = 76^\circ$ (2sf)

4. a) $T = 2 \times 3 = 6s$ $\quad a = 2L$ $\quad \omega = \frac{2\pi}{T} = \frac{\pi}{3}$

$x = a\cos\omega t$

$(2L - b) = 2L\cos\frac{\pi}{3} \times \frac{3}{4}$

$2L - b = 2L\cos\frac{\pi}{4}$

$2L - b = 2L\frac{\sqrt{2}}{2}$

$b = 2L - \sqrt{2}L$

$= (2 - \sqrt{2})L$

b) $v^2 = \omega^2(a^2 - x^2)$

$v^2 = \frac{\pi^2}{9}\left(4L^2 - (\sqrt{2}L)^2\right) = \frac{\pi^2}{9} \cdot 2L^2$

$v = \frac{\sqrt{2}}{3}\pi L$

c) $x = a\sin\omega t$

$\frac{1}{2}(2-\sqrt{2})L = 2L\sin\frac{\pi}{3}t$

$2-\sqrt{2} = 4\sin\frac{\pi}{3}t$

$\frac{2-\sqrt{2}}{4} = \sin\frac{\pi}{3}t$

$\frac{\pi}{3}t = 0.1470s$

$t = 0.1404s$

Time taken $= 0.28s$ (2dp)

5. a) $a = -\frac{3}{\sqrt{t+4}}$

$$\frac{dv}{dt} = -3(t+4)^{-\frac{1}{2}}$$

$$\int_{18}^{v} dv = -3\int_{0}^{t} (t+4)^{-\frac{1}{2}}\,dt$$

$$\left[v\right]_{18}^{v} = -\cancel{3}2\left[(t+4)^{\frac{1}{2}}\right]_{0}^{t}$$

$$v - 18 = -6\left((t+4)^{\frac{1}{2}} - 2\right)$$

$$v = 12 + 18 - 6\sqrt{t+4} = \left(30 - 6\sqrt{t+4}\right)\,\text{ms}^{-1}$$

b) $0 = 30 - 6\sqrt{t+4}$

$5 = \sqrt{t+4}$

$25 = t+4$

$t = 21\text{s}$

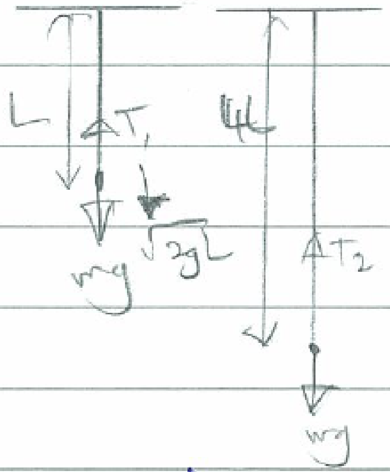
$$\frac{dx}{dt} = 30 - 6\sqrt{t+4}$$

$$\int_{0}^{x} dx = \int_{0}^{21} 30 - 6(t+4)^{\frac{1}{2}}\,dt$$

$$x = \left[30t - 4(t+4)^{\frac{3}{2}}\right]_{0}^{21}$$

$$= 130 + 32 = 162\,\text{m}$$

6.



a) $\frac{1}{2}\cdot m\cdot\left(\sqrt{2gL}\right)^2 = \frac{\lambda\cdot(3L)^2}{2L} - mg\cdot 3L$

$\frac{1}{\cancel{2}}\cancel{m}\cancel{2}gL = \frac{\lambda 9L^{\cancel{2}}}{2\cancel{L}} - 3\cancel{L}mg$

$2mg + 6mg = 9\lambda$

$\lambda = \frac{8mg}{9}$

b) $T = \frac{\lambda e}{a}$

$mg = \frac{8mg}{9L}\cdot e$

$9L = 8e$

$e = \frac{9L}{8}$

At P, $[F = ma]$

$mg - \frac{8mg}{9}\left(\frac{9L}{8} + x\right)\cdot\frac{1}{L} = m\ddot{x}$

$g - g - \frac{8}{9L}gx = \ddot{x}$

$\ddot{x} = -\frac{8g}{9L}x$

c) i) $T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\frac{8g}{9L}}} = \frac{2\pi\cdot 3\sqrt{L}}{2\sqrt{2g}} = 3\pi\sqrt{\frac{L}{2g}}$

ii) $v_{max} = a\omega = \frac{15L}{8} \times \sqrt{\frac{8g}{9L}} = \frac{\cancel{15}L}{8L} \times \frac{2\sqrt{2g}}{\cancel{3}\sqrt{L}} = \frac{5}{4}\sqrt{2g}$

7- a) $\frac{1}{2}m \cdot 15 + mg(5 - 5\cos 60) = \frac{1}{2}\cancel{m}v^2$

$v^2 = 15 + 5g = 64$

$v = 8\ ms^{-1}$

b) $\cancel{\frac{1}{2} \cdot 60 \times 8^2 + \frac{1}{2}m \times 3^2 - (60+m)g \times 2.5}$

$60 \times 8 - 3m = (60+m)v$ $\cancel{30 \times 64 + \frac{9}{2}m = 1470 + 24.5m}$

$v = \frac{480 - 3m}{60 + m}$ $\cancel{20m = 450}$ $\cancel{m = 22.5}$

$\frac{1}{2}(60+m)\frac{(480-3m)^2}{(60+m)^2} = (60+m) 2.5g$

$480^2 - 2880m + 9m^2 = (60+m)^2 \times 49$

$480 - 3m = (60+m)7$, $10m = 60$, $m = 6kg$

c) $v = \frac{480 - 18}{66}$

$= 7\ ms^{-1}$

$T - 66g = \frac{66 \times 49}{5}$

$T = 1294N$